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# THE EFFECTS OF AXIAL CONDUCTION IN THE WALL ON HEAT TRANSFER WITH LAMINAR FLOW

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**Abstract**—Wall conduction effects on steady-state laminar flow heat-transfer experiments are examined, and an analysis of heat transfer with axial conduction in the wall bounding a fluid in laminar flow is developed to determine the effects of the conduction in the wall on heat transfer with Poiseuille–Couette flow between parallel plates. The parameters that determine the relative importance of axial conduction are found to be the Péclét number of the fluid, the thickness to length ratio of the wall and the parameter $\beta = k_2\delta_1/k_1L$. The Couette flow analysis and experiments that correspond to heat transfer with Couette flow are shown to be in good agreement, and comparisons of the interfacial temperature distribution and local Nusselt numbers obtained by accounting for axial conduction are made with the results determined by neglecting axial conduction.

## NOMENCLATURE

| | |
|---|---|
| $a_n$, | parameter introduced in equation (10); |
| $A_n$, $A_n^*$, | eigenconstants; |
| $J_{\frac{1}{3}}(z)$, $J_{-\frac{1}{3}}(z)$, | Bessel functions; |
| $k$, | thermal conductivity [Btu/hft°F]; |
| $K_{1_n}$, $K_{2_n}$, | eigenconstants; |
| $L$, | heated length [ft]; |
| $M(a, b, z)$, | Kummer's function; |
| $Pe$, | Péclét number for the fluid phase; |
| $q$, | heat flux [Btu/hft$^2$]; |
| $S$, | dimensionless parameter; |
| $T$, | temperature [°F]; |
| $u$, $U$, | velocities [ft/h or ft/s]; |
| $V$, | temperature; |
| $x$, | distance from the leading edge; |
| $y$, | distance from the wall; |
| $z_n$, | independent variable in equation (10). |

Greek letters

| | |
|---|---|
| $\alpha$, | thermal diffusivity [ft$^2$/s]; |
| $\beta$, | dimensionless parameter; |
| $\gamma_n$, | eigenvalue; |
| $\delta$, | thickness [ft]; |
| $\varepsilon_n$, | parameter introduced in equation (14); |
| $\zeta$, | dimensionless independent variable; |
| $\eta$, | dimensionless independent variable; |
| $\theta$, | dimensionless temperature; |
| $\lambda_n$, $\lambda_n^*$, | eigenvalues; |
| $\xi$, | dimensionless independent variable; |
| $\tau$, | coefficients in equation (22); |
| $\phi$, | function introduced in equation (23); |
| $\Psi_n$, | eigenfunction; |
| $\omega_n$, | parameter in equation (23). |

Subscripts

| | |
|---|---|
| $e$, | refers to a condition at the leading edge; |
| $m$, | refers to the $m$th eigenfunction or eigenconstant or a mixed mean temperature; |
| max, | refers to the maximum velocity for plane Poiseuille flow; |
| $n$, | refers to the $n$th eigenfunction or eigenconstant; |

| | |
|---|---|
| 0, | refers to a condition at the solid–fluid interface; |
| $w$, | refers to a constant wall boundary condition; |
| 1, | refers to the fluid phase; |
| 2, | refers to the solid phase (wall). |

## INTRODUCTION

In the design and analysis of heat exchange equipment and in the interpretation of experimental data, axial conduction in the wall bounding a fluid is usually ignored, but it can have a significant effect on the heat transfer and temperature field in the fluid adjacent to the wall. This is especially true in the thermal entrance region. Although the problem is similar to heat transfer in a composite body, the phenomenon has been subjected to very little analysis. In the most significant related paper Perelman [1] calls this type of problem a "conjugated" boundary value problem, and he examined two problems of heat transfer to a fluid flowing around a body containing internal heat sources. In addition he considered the asymptotic solutions to the types of integral equations that occur in the analysis of such conjugated problems. He treated the relatively simple flow configurations of slip flow around a body and laminar boundary layer flow over a thin plate, but his work is an excellent summary and analysis of the basic problem. Sell and Hudson [2] considered the effect of wall conduction on heat transfer to slug flow, and Rotem [3] developed an approximation method for determining the wall temperature profile and the heat transfer coefficient for heat transfer to a laminar boundary layer with conduction in the wall. Rotem's method, however, applies to systems for which the wall boundary condition is either approximately constant temperature or approximately constant heat flux.

Schenk *et al.* [4, 5] and Sideman *et al.* [6] studied a problem that is somewhat related to the present problem. The former investigators extended the Grätz problem for flow between parallel plates to include the effects of surface resistance to heat transfer, and the latter investigators extended and supplemented the results of Schenk *et al.* by solving the problem for both circular and flat conduits. The essential difference between the present problem and the Grätz problem with constant surface resistance is that no *a priori* information about the surface resistance is assumed here. Thus the problem is similar to the class of conjugated boundary value problems discussed by Perelman.

Recently Gill *et al.* [7] have shown how the solutions for the temperature profile for single-stream forced convection heat transfer problems can be used to construct the temperature distribution in the thermal entrance region of multistream concurrent flow heat exchangers. This approach, which will be used in the present study, can be applied to problems involving a fluid stream and a solid boundary for arbitrary interfacial temperature or heat flux distribution. Davis and Cooper [8], in studies of heat transfer to thin liquid film flow, conjectured (from analysis of their theoretical and experimental results) that axial conduction substantially affected their heat transfer results in the thermal entrance region.

It is the purpose of this paper to show how the effects of axial conduction in the wall can be predicted from the solutions for the temperature fields in the liquid phase and solid wall considered separately and to use the analysis to interpret the results of Davis and Cooper. Although the analysis is developed for the problem of Poiseuille–Couette flow between parallel planes with no heat generation in the wall, it can readily be extended to other geometries, and heat generation in the wall can be treated.

## PROBLEM FORMULATION

Consider laminar flow between parallel planes, shown schematically in Fig. 1. If the upper surface of the fluid moves with velocity $U$ (either because of motion of the solid boundary or because of the shearing flow of a gas) and if a pressure gradient exists, the velocity profile for this combined Poiseuille–Couette flow can be written as

$$u = 4u_{\max}(S\zeta - \zeta^2) \tag{1}$$

where $S = 1 + U/4u_{\max}$ and $u_{\max}$ is the maximum velocity for a flow with the same pressure gradient but with stationary boundaries (Poiseuille flow). The velocity profile given by equation (1), then, reduces to that for Poiseuille flow when $U = 0$ and to that for Couette flow in the absence of a pressure gradient.

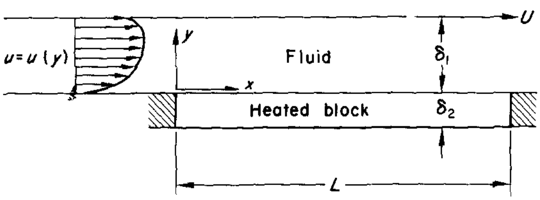


FIG. 1. The system under consideration.

For the experimental conditions of interest in the present work axial conduction in the fluid (Hennecke [9] has shown that for Hagen–Poiseuille flow axial conduction is important only for small Péclét numbers) and viscous heat dissipation can be ignored and the fluid properties are constant; therefore the temperature field in the fluid is described by

$$u\frac{\partial T_1}{\partial x} = \frac{\partial^2 T_1}{\partial y^2}. \tag{2}$$

The subscript 1 will be used to denote the temperature and properties of the fluid phase, and the subscript 2 will refer to the wall.

For two-dimensional steady state conduction in an isotropic medium with no internal heat generation the temperature field in the wall bounding the flow is given by

$$\frac{\partial^2 T_2}{\partial x^2} + \frac{\partial^2 T_2}{\partial y^2} = 0. \tag{3}$$

The assumptions apply to a wall heated from the lower side by some external source (a condensing vapor, electrical source, etc.), but the problem involving heat generation in the wall also can be treated by the method discussed here [replacing equation (3) by the appropriate energy equation, of course].

Equations (2) and (3) are coupled by continuity conditions at the solid–fluid interface:

$$T_1(x, 0) = T_2(x, 0) = T_0(x) \tag{4}$$

and

$$k_1\frac{\partial T_1}{\partial y}(x, 0^+) = k_2\frac{\partial T_2}{\partial y}(x, 0^-). \tag{5}$$

Numerous combinations of boundary conditions at the upper surface of the fluid and the lower surface of the wall are of some interest, but to illustrate the method of analysis and to compare the analysis with available experimental data we shall consider the following boundary conditions in addition to the fluid–solid interfacial condition:

(i) $T_1(0, y) = T_e =$ constant (the thermal entry condition)
(ii) $T_1(x, \delta_1) = T_e$
(iii) $\dfrac{\partial T_2}{\partial x}(0, y) = 0$
(iv) $\dfrac{\partial T_2}{\partial x}(L, y) = 0$
(v) $\dfrac{\partial T_2}{\partial y}(x, -\delta_2) = -\dfrac{q_w}{k_2}$ (a constant heat flux at the lower surface)

The continuity conditions at the fluid–solid interface, equations (4) and (5), represent the other boundary conditions that must be satisfied.

These boundary conditions are consistent with the experimental facility of Davis and Cooper, which is discussed below. Numerous other combinations of boundary conditions could be involved experimentally, but other boundary conditions may be treated merely as variations on the theme developed here.

Since it will be shown that the solution for the coupled equations, (2) and (3), can be written in terms of the solutions for the temperature fields in the individual phases considered separately, the application of the method to other boundary conditions is readily carried out. To this end we shall first obtain solutions to the appropriate differential equations for an arbitrary unknown interfacial temperature distribution by using the Duhamel theorem. Then, the interfacial condition that satisfies the partial differential equations and the boundary conditions is obtained by applying the continuity conditions.

### THE FLUID PHASE TEMPERATURE FIELD

The temperature field in the fluid phase for a variable interfacial temperature can be written in terms of the solution for a constant interfacial condition by applying the extended Duhamel theorem discussed by Bartels and Churchill [10]. It is the interfacial temperature distribution that is of interest, so first we solve the problem with a constant temperature boundary condition and then generalize this to the solution for arbitrary interfacial temperature. For a constant wall temperature the differential equation is

$$u\frac{\partial V}{\partial x} = \alpha\frac{\partial^2 V}{\partial y^2} \tag{6}$$

where $u = u(y)$, given by equation (1), $V = V(x, y)$, and the boundary conditions are:

(a) $V(0, y) = T_e$
(b) $V(x, \delta_1) = T_e$
(c) $V(x, 0) = T_0 =$ constant.

Because this problem is closely related to the well-known Grätz problem and its various extensions there is little need to present a detailed account of its solution, but a brief recapitulation of the solution is in order here because the analysis represents a more general form of the problem. The Grätz problem appears as the special case when $S = 1$ in the velocity profile expression.

Substituting equation (1) into equation (6) and introducing the dimensionless variables

$$\theta = \frac{(V - T_e)}{(T_0 - T_e)}, \quad \zeta = \frac{y}{\delta_1}, \quad \eta = \frac{x}{\delta_1 Pe(S - \frac{2}{3})},$$

where the Péclét number for the liquid is defined by $Pe = 4u_{max}\delta_1/\alpha$, gives the dimensionless equation

$$\frac{\partial^2\theta}{\partial\zeta^2} = \frac{(S\zeta - \zeta^2)}{(S - \frac{2}{3})}\frac{\partial\theta}{\partial\eta}. \tag{7}$$

The term $(S - \frac{2}{3})$ is introduced as a convenient normalizing factor for the velocity distribution. Equation (7) was solved in the usual way to give

$$\theta(\eta, \zeta) = 1 - \zeta + \sum_{n=1}^{\infty} A_n\Psi_n(\zeta)\exp(-\lambda_n^2\eta). \tag{8}$$

The eigenvalues $\lambda_n$ and the eigenfunctions $\Psi_n(\zeta)$ satisfy

$$\frac{d^2\Psi}{d\zeta^2} + \lambda^2\frac{(S\zeta - \zeta^2)}{(S - \frac{2}{3})}\Psi = 0. \tag{9}$$

The solution of equation (9) is conveniently written in terms of the confluent hypergeometric function to give

$$\Psi_n(\zeta) = \exp\left(-\frac{z_n}{2}\right)[K_{1_n}M(a_n, \tfrac{1}{2}, z_n) + K_{2_n}z_n^{\frac{1}{2}}M(a_n + \tfrac{1}{2}, \tfrac{3}{2}, z_n)] \tag{10}$$

where

$$z_n = \lambda_n^*\left(\zeta - \frac{S}{2}\right)^2, \quad a_n = -\frac{S^2\lambda_n^*}{16} + \tfrac{1}{4},$$

and

$$\lambda_n^* = \frac{\lambda_n}{(S - \frac{2}{3})^{\frac{1}{2}}}.$$

The constants $K_{1_n}$ and $K_{2_n}$ are obtained by applying the boundary conditions, and the eigenconstants $A_n$ in equation (8) are obtained by applying the thermal entry condition in the usual way. Epton has tabulated the eigenconstants and eigenvalues for this problem for various values of the parameter $S$ [11].

Of particular interest to the authors is the problem involving Couette flow of the fluid phase. In this case equation (1) reduces to a linear profile, and the solution of equation (6) can be obtained by considering the asymptotic solution of equation (7) as $S \to \infty$. The solution is of the same form as equation (8), but $\Psi_n(\zeta)$

can be written either in terms of the Airey function or the Bessel function. The Bessel function solution is

$$\Psi_n(\zeta) = \lambda_n^{\frac{1}{3}}\zeta^{\frac{1}{2}}J_{\frac{1}{3}}(\tfrac{2}{3}\lambda_n\zeta^{\frac{3}{2}}) \tag{11}$$

and the eigenvalues satisfy

$$J_{\frac{1}{3}}(\tfrac{2}{3}\lambda_n) = 0. \tag{12}$$

Using the above results the solution for the constant interfacial temperature problem becomes

$$V(x, y) = T_e + (T_0 - T_e) \times \left[1 - \frac{y}{\delta_1} + \sum_{n=1}^{\infty} A_n\Psi_n\left(\frac{y}{\delta_1}\right) \times \exp\left(-\frac{\lambda_n^{*2}x}{\delta_1 Pe}\right)\right] \tag{13}$$

The solution for the problem with a variable interfacial temperature can be written by applying Duhamel's theorem to the above solution to give

$$T_1(x, y) = T_e + \frac{\partial}{\partial x}\int_0^x \Big\{[T_0(x') - T_e] \times \{1 - \frac{y}{\delta_1} + \sum_{n=1}^{\infty} A_n\Psi_n\left(\frac{y}{\delta_1}\right) \times \exp[-\varepsilon_n(x - x')]\}\Big\}\, dx' \tag{14}$$

where $\varepsilon_n = \lambda_n^{*2}/\delta_1 Pe$ (for Couette flow $\varepsilon_n = \lambda_n^2/\delta_1 Pe$).

The heat flux at the interface is given by

$$-k_1\frac{\partial T_1}{\partial y}(x, 0^+) = \frac{k_1}{\delta_1}\frac{\partial}{\partial x}\int_0^x \Big\{[T_0(x') - T_e] \times \{1 - \sum_{n=1}^{\infty} A_n^* \exp[-\varepsilon_n(x - x')]\}\Big\}\, dx' \tag{15}$$

where $A_n^* = A_n\Psi_n'(0)$ and the prime indicates differentiation with respect to $y$.

### THE SOLID PHASE TEMPERATURE FIELD

The temperature distribution in the wall can be obtained by solving equation (3) subject to boundary conditions (iii), (iv), (v) and

$$T_2(x, 0) = T_0(x) = f(x). \tag{16}$$

If $T_0(x)$ is assumed to be known, which it obviously is not at this point, then the solution can be obtained by the classical methods discussed by Carslaw and Jaeger [12] to give

$$T_2(x, y) = -\frac{q_w}{k_2}y + \frac{2}{L}\sum_{m=1}^{\infty} C_m \frac{\cosh\gamma_m(y + \delta_2)}{\cosh\gamma_m\delta_2}\cos\gamma_m x \tag{17}$$

where the Fourier coefficients $C_m$ are given by

$$C_m = \int_0^L T_0(x)\cos\gamma_m x\, dx \tag{18}$$

and the eigenvalues $\gamma_m$ are

$$\gamma_m = \frac{m\pi}{L}. \tag{19}$$

In this case the interfacial flux becomes

$$-k_2\frac{\partial T}{\partial y}(x, 0) = q_w - \frac{2k_2}{L}\sum_{m=1}^{\infty} C_m\gamma_m \tanh\gamma_m\delta_2\cos\gamma_m x. \tag{20}$$

Equations (14) and (17), the solutions for the temperature distribution in the liquid and solid phases respectively, both contain the unknown $T_0(x)$, and to complete the solution one must now find this interfacial temperature distribution.

Although the solid phase temperature distribution is developed above for a finite length $L$ because of the application in mind, it is possible to modify the analysis for a heated region of infinite length by using a Green's function or Fourier transform approach.

## THE CONJUGATED PROBLEM SOLUTION

By applying the continuity condition for the heat flux, equation (5), and multiplying the resulting equation by $\delta_1/k_1$ we obtain equation (21), which is the integro-differential equation that must be solved to obtain the interfacial temperature distribution,

$$\frac{\partial}{\partial x}\int_0^x \Big\{[T_0(x') - T_e]\{1 - \sum_{n=1}^{\infty} A_n^* \exp[-\varepsilon_n(x - x')]\}\Big\}\,dx' = \frac{q_w\delta_1}{k_1} - 2\beta \sum_{m=1}^{\infty} C_m\gamma_m \tan\gamma_m\delta_2 \cos\gamma_m x. \qquad (21)$$

The parameter $\beta$ is the ratio of thermal resistance across the liquid film transversely to that in the solid over its entire axial length, and it is given by

$$\beta = k_2\delta_1/k_1 L.$$

Qualitatively, when $\beta$ is small axial conduction has little effect on the interfacial temperature distribution compared with the solution for a constant specified wall flux.

Equation (21) is conveniently solved by assuming a temperature distribution at the interface of the form

$$T_0(x) = \tau_0 + \tau_1 x + \tau_2 x^2 + \tau_3 x^3 + \ldots. \qquad (22)$$

The substitution of equation (22) in equation (21) followed by the appropriate integration and differentiation leads to an equation of the form

$$\tau_0\phi_0(x) + \tau_1\phi_1(x) + \tau_2\phi_2(x) + \ldots = T_e\phi_0(x) - \frac{q_w\delta_1}{k_1} \qquad (23)$$

where

$$\phi_0(x) = -1 + \sum_{n=1}^{\infty} A_n^* \exp(-\varepsilon_n x)$$

$$\phi_1(x) = -x + \sum_{n=1}^{\infty} A_n^*\omega_n - \frac{2\beta L}{\pi}\sum_{m=1}^{\infty}\frac{[(-1)^m - 1]}{m}\tanh\gamma_m\delta_2\cos\gamma_m x$$

$$\phi_2(x) = -x^2 + \sum_{n=1}^{\infty} A_n^*(x - \omega_n)/\varepsilon_n - \frac{4\beta L^2}{\pi}\sum_{m=1}^{\infty}\frac{(-1)^m}{m}\tanh\gamma_m\delta_2\cos\gamma_m x$$

$$\phi_j(x) = -x^j + \sum_{n=1}^{\infty} A_n^*\frac{\partial}{\partial x}\int_0^x (x')^j \exp[-\varepsilon_n(x - x')]\,dx' - 2\beta\sum_{m=1}^{\infty}\left(\int_0^L x^j\cos\gamma_m x\,dx\right) \times \gamma_m\tanh\gamma_m\delta_2\cos\gamma_m x$$

and

$$\omega_n = [1 - \exp(-\varepsilon_n x)]/\varepsilon_n.$$

A collocation technique was used to determine the coefficients $\tau_0$, $\tau_1$, $\tau_2$, ... to obtain the interfacial temperature distribution, i.e. by writing equation (23) for various values of $x$, say $x_1$, $x_2$, $x_3$, ..., a system of linear simultaneous equations is obtained which can be solved to evaluate the coefficients.

## EXPERIMENTS

Because experimental data are available for a system that conforms to the phenomenon and boundary conditions considered above it is possible to compare the analysis with the experimental data. Although the experimental equipment and techniques are reported elsewhere [8, 13], a brief discussion is in order here.

Davis and Cooper studied the flow of a liquid film dragged over a heated surface by a concurrent gas or vapor flow, and they made heat-transfer measurements in the thermal entrance region. If the film flow is not unstable the asymptotic case of Couette flow is closely approximated, and many of the experiments reported involved smooth film flow.

The experimental facility, shown in [8], consisted of a 20 ft long wind tunnel, 10 in. wide and 1 in. high. A liquid film was introduced near the air inlet through perforations in the bottom of the tunnel, and a 2 ft long heat-transfer plate, consisting of a 1 in. thick copper block extending the width of the tunnel, was installed about 16 ft from the air inlet. The heat-transfer test section was sufficiently far downstream from the air and water inlets to insure fully developed flow in the downstream portion of the wind tunnel. Electrical strip heaters, attached to the bottom of the copper plate, were used to supply heat to the system. The power to the 500 W strip heaters, wired in banks of four heaters per bank, was controlled by means of variable transformers and was measured with calibrated voltmeters and ammeters. The upstream and downstream edges of the block were insulated and butted against Plexiglass, and a backup heater was installed beneath the primary heaters to minimize heat loss. Both constant heat flux, corresponding to boundary condition (v.), and constant wall temperature experiments were conducted, but because the constant heat flux runs showed the more pronounced effects of axial conduction in the block, they are of particular interest here.

To obtain the wall temperature (interfacial temperature) and the local heat flux eight sets of thermocouples were installed in the copper block at eight axial stations. From information on the vertical temperature profile in the block at each station the surface temperature was obtained by extrapolation. The various fluid dynamics parameters required (liquid surface velocity, film thickness, entry temperature, etc.) were measured as discussed in [8]. Table 1 is a summary of the significant parameters involved. It is to be noted that the parameter $\beta$ is of the order of one, suggesting that axial conduction is important.

*Table 1. The experimental parameters*

| Run | $\beta$ | $L/\delta_1 Pe$ | $\delta_1$ (ft) | $Pe$ | $q_w$ (Btu/hft$^2$) | $T_e$ (°F) |
|---|---|---|---|---|---|---|
| 14 | 0·942 | 0·234 | 0·00330 | 2590 | 4680 | 65·7 |
| 17 | 1·306 | 0·473 | 0·00237 | 1780 | 5020 | 61·9 |
| 19 | 0·765 | 0·142 | 0·00402 | 3510 | 8890 | 61·3 |
| 22 | 0·970 | 0·251 | 0·00317 | 2520 | 8940 | 61·0 |
| 26 | 1·205 | 0·508 | 0·00255 | 1540 | 8950 | 63·5 |
| 27 | 1·312 | 0·555 | 0·00236 | 1520 | 9030 | 64·1 |

$\delta_2/L = 0{\cdot}04167$

## RESULTS

To elucidate quantitatively the role of the various parameters influencing the importance of axial conduction in the wall a parametric study was carried out on the digital computer. Inspection of equation (21) indicates that the Péclét number of the liquid, $Pe$, the dimensionless parameter $\beta$ and the ratio $\delta_2/L$ determine the relative importance of the axial conduction. Since heat will flow most rapidly along the path of least resistance, qualitatively we can expect axial conduction in the copper block to be diminished by anything which contributes to increasing the rate of heat transfer transversely across the block and the flow. Consequently since $\beta$ is the ratio of the transverse resistance in the flow to the axial resistance in the block, increasing this parameter will increase the importance of axial conduction in the block. Increasing the Péclét number decreases the thermal resistance across the flow and therefore this decreases the effect of axial conduction. Lastly, $\delta_2/L$ can be viewed as the ratio of the transverse to axial thermal resistance in the block and therefore increasing this parameter should increase the importance of axial conduction. Conversely, it follows that an increase in axial conduction will detract from transverse rates of transfer and is therefore usually a deleterious effect.

Figures 2 and 3 show the effect on the interfacial temperature distribution of varying the Péclét number for $\beta = 0{\cdot}1$ and $\beta = 10$, respectively for a fairly large length to thickness ratio (100:1). The former figure shows that as the Péclét number increases the effects of axial conduction in the wall become insignificant, and the limiting interfacial temperature profile predicted for a constant wall heat flux is approached When $\beta$ is large, however, axial conduction in the wall greatly effects the interfacial temperature distribution and the heat flux through the wall. Figure 3 shows that at even relatively large values of the Péclét number the constant heat

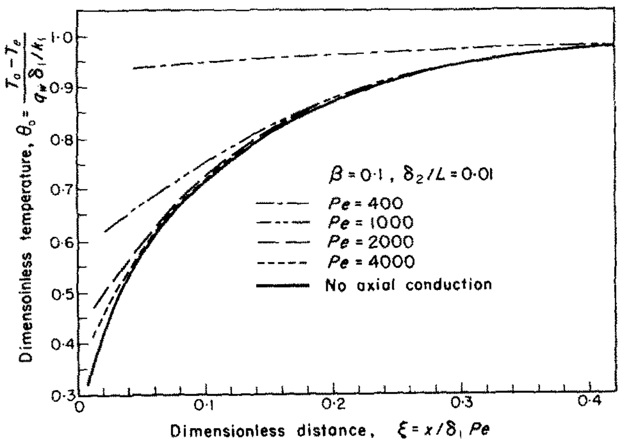


FIG. 2. The effect of axial conduction in the wall on the interfacial temperature profile for small $\beta$ and for various Péclét numbers.

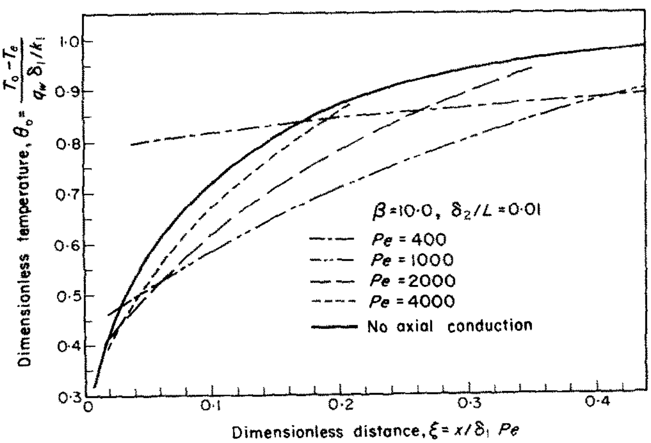


FIG. 3. The effect of axial conduction in the wall on the interfacial temperature profile for large $\beta$ and for various Péclét numbers.

flux temperature distribution is not attained. The curve for a Péclét number of 400 in Fig. 3 approaches the curve for no axial conduction only for values of the dimensionless distance from the leading edge that are much greater than the range of the figure. For a specified Péclét number and $\beta$ the thickness to length ratio of the wall has the effect on the interfacial temperature shown in Fig. 4. As expected, axial conduction in the wall is less significant as the wall thickness decreases.

Equation (22) was found to be a rapidly convergent series, for the order of the polynomial selected had no apparent effect on the solution for polynomials of second order or greater, but the results presented here are for third order polynomials.

Figures 5 and 6 compare the analysis neglecting axial conduction in the wall, the present analysis and the experimental data of Davis and Cooper. The effect of axial conduction in the wall is to increase the temperature near the

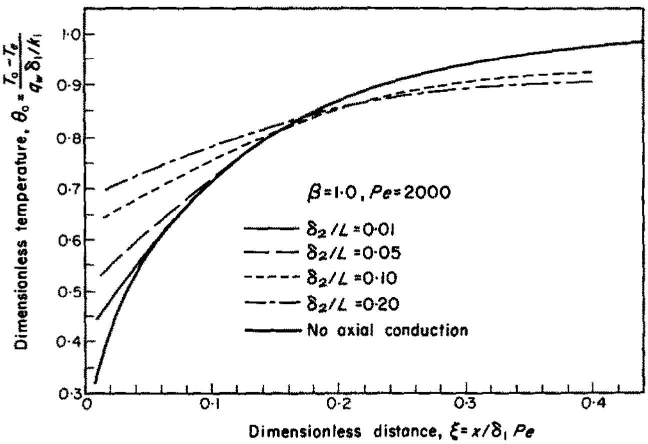


FIG. 4. The effect of axial conduction in the wall on the interfacial temperature profile for various values of $\delta_2/L$.

In the calculations for Figs. 2–4 and those that follow it was found that the collocation techniques used to obtain the coefficients of equation (21) was economical of computer time, but the coefficients and the resulting interfacial temperature profiles depend slightly on the collocation points selected for the evaluation of the $\phi_j(x)$ functions. The curves shown in Figs. 2–6 should actually be considered to be narrow bands, the maximum and minimum deviations from the lines shown being approximately 2 per cent. Within these limits the collation points could be varied considerably, for the calculations were primarily affected by the collocation point nearest the leading edge.

leading edge over that predicted for a constant heat flux at the interface, and to lower the temperature slightly in the downstream portion. Differences between the analysis and experimental data can be attributed to two effects. First, there are experimental errors involved as indicated in [8] and second, the analysis does not account for small heat losses through the ends of the block. The results of the present analysis are in reasonably good agreement with the measured interfacial temperatures. Somewhat poorer agreement is obtained in a comparison of the predicted and measured local Nusselt numbers, however, for the Nusselt numbers calculated from the experimental data

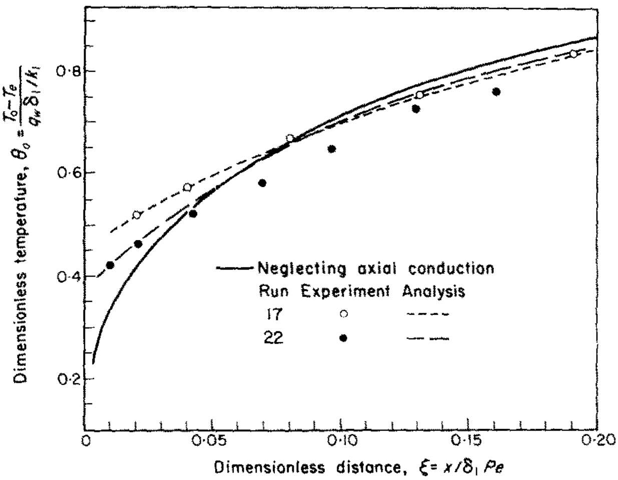


FIG. 5. A comparison of predicted interfacial temperature profiles with experimental data.

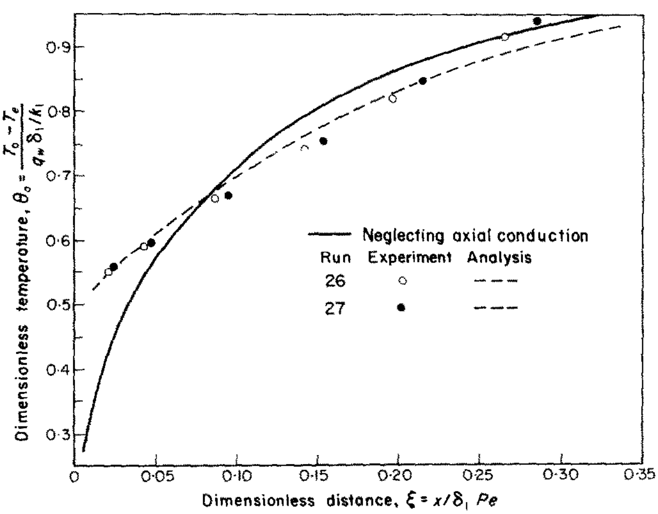


FIG. 6. A comparison of predicted interfacial temperature profiles with experimental data.

are more subject to errors in the measurements than are the dimensionless temperature profiles. Figure 7 is a comparison among the experimental results and the Nusselt numbers predicted by

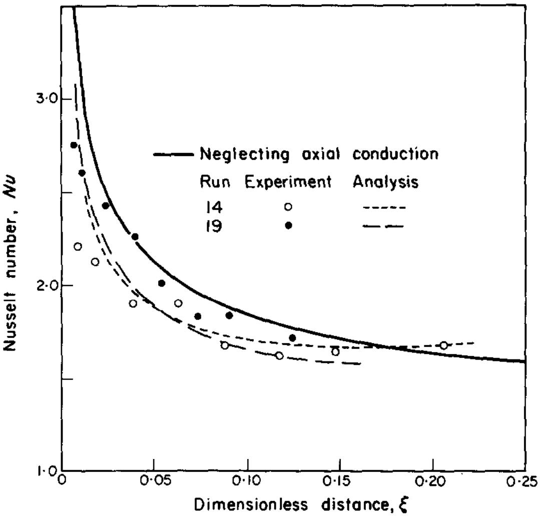


FIG. 7. A comparison of predicted Nusselt numbers with experimental data.

neglecting and by accounting for the axial conduction. The Nusselt number for the thin film flow may be defined by

$$Nu = -\frac{\delta_1 \left.\frac{\partial T}{\partial y}\right|_{y=0}}{(T_0 - T_m)} = \frac{q_0 \delta_1}{k_1(T_0 - T_m)} \quad (24)$$

where the subscript $m$ refers to the mixed mean temperature and $q_0$ is the local heat flux at the solid–fluid interface. When a constant heat flux, $q_0 = q_w =$ constant, is involved, i.e. negligible axial conduction, the Nusselt number can be written in terms of the appropriate dimensionless temperature for that case, $\theta = (T - Te)/(q_w \delta_1/k_1)$, to give

$$Nu = 1/(\theta_0 - \theta_m). \quad (25)$$

The effect uf the axial conduction, shown in Fig. 7, is to lower the Nusselt numbers relative to those predicted for a constant heat flux at the solid–fluid interface. For the relatively short heat-transfer section of Davis and Cooper the asymptotic value of the Nusselt number of 1·5 was not attained.

## CONCLUSIONS

Axial conduction in the wall of a heat-transfer apparatus can significantly affect the temperature field in the fluid phase and lower the Nusselt number associated with the heat transfer. The Péclét number of the fluid, $Pe$, the thickness to length ratio of the wall, $\delta_2/L$, and the dimensionless group of variables, $\beta = k_2\delta_1/k_1L$, are the important parameters in determining the effects of the axial conduction. Increasing $\beta$ and $\delta_2/L$ increases the importance of axial conduction whereas increasing $Pe$ decreases it.

The analysis of axial conduction in the wall developed above and applied to the system studied by Davis and Cooper shows that the experimental deviations of the wall temperature profile from the profile predicted for the constant heat flux boundary condition are in large measure due to axial conduction in the test section wall.

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## LES EFFETS DE LA CONDUCTION AXIALE DANS LA PAROI SUR LE TRANSPORT DE CHALEUR AVEC UN ÉCOULEMENT LAMINAIRE

**Résumé**—Les effets de la conduction de la paroi sur des expériences de transport de chaleur en écoulement laminaire et en régime permanent sont examinés, et une analyse du transport de chaleur avec conduction axiale dans la paroi limitant un fluide en écoulement laminaire est élaborée pour déterminer les effets de la conduction dans la paroi sur le transport de chaleur avec un écoulement de Poiseuille–Couette entre des plaques parallèles. Les paramètres qui déterminent l'importance relative de la conduction axiale sont le nombre de Péclet du fluide, le rapport épaisseur sur longueur de la paroi et le paramètre $- K_2\delta_1/k_1L$. L'analyse de l'écoulement de Couette et les expériences qui correspondent au transport de chaleur avec l'écoulement de Couette sont en bon accord, et la distribution de température interfaciale et les nombres de Nusselt locaux obtenus en tenant compte de la conduction axiale sont comparés avec les résultats déterminés en négligeant la conduction axiale.

## DER EINFLUSS ACHSIALER WANDBEDINGUNGEN AUF DEN WÄRMEÜBERGANG BEI LAMINARSTRÖMUNG

**Zusammenfassung**—Es wird der Einfluss der Wärmeleitung in der Wand auf die Versuche mit stationärem Wärmeübergang bei laminarer Strömung untersucht. Es wird eine Berechnungsmethode für den Wärmeübergang mit axialer Leitung in den Wänden, die eine laminar strömende Flüssigkeit begrenzen, entwickelt, um den Einfluss der Leitung in der Wand auf den Wärmeübergang bei Poiseuille-Couette-Strömung zwischen parallelen Platten zu bestimmen. Als Parameter, welche die relative Bedeutung der axialen Leitung ausdrücken, wurden die Peclet-Zahl der Flüssigkeit, das Verhältnis der Dicke zur Länge der Wand und der Parameter $K_2\,\delta_1/k_1L$ gefunden. Die Berechnungsmethode der Couette-Strömung und die Versuche, die dem Wärmeübergang bei Couette-Strömung entsprechen, zeigen gute Übereinstimmung.

Die Temperaturverteilung zwischen den Wänden und die lokalen Nusselt-Zahlen bei Berücksichtigung der axialen Leitung werden verglichen mit Ergebnissen bei denen die axiale Leitung vernachlässigt wurde.

## ВЛИЯНИЕ ОСЕВОГО УСЛОВИЯ В СТЕНКЕ НА ТЕПЛООБМЕН В ЛОКАЛРНОМ ПОТОКЕ

**Аннотация**—Экспериментально исследуются влияния теплопроводности стенки на стационарный теплообмен при ламинарном течении. Для определения влияния теплопроводности в стенке на теплообмен при наличии пуазейлевского течения и течения Куэтта между параллельными пластинами анализируется теплообмен при наличии аксиального кондуктивного теплового потока теплопроводности в стенке, ограничивающей ламинарный поток жидкости. Найдено, что параметрами, определяющими относительное влияние аксиального теплового потока за счет теплопроводности являются числр Пекле для жидкости, отношение толщины стенки к её длине и параметр $= K_2\delta_1/K_1L$. Показано, что теоретические данные по теплообмену в куэттовском потоке хорошо согласуются с экспериментальными. Данные по распределению температуры в области межфазной границы и для локальных чисел Нуссельта с учетом теплопроводности стенки сравниваются с результатами, полученными без учета кондуктивного теплового потока в осевом направлении.